Name:_____ Date:_____

Math 12H HW 4.8: Half Angle Identities and Solving advanced trigonometric equations

Half Angle Identities

$$\sin\tfrac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\tfrac{1}{2}\theta = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\tfrac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

Sum of Angle properties:

$$R\cos(\theta-\alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha \qquad R\sin(\theta-\alpha) = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$$

1. Use the half angle identities to evaluate each of the following:

a) $\sin 15°$	b) $\cos 22.5°$
c) $\sin 202.5°$	d) $\cos 202.5°$
e) $\sin 292.5°$	

2. Solve for θ, where $0° \le \theta \le 360°$ by using the form $R\cos(\theta-\alpha)$ *or* $R\sin(\theta-\alpha)$

a) $3\cos\theta + 2\sin\theta = 1$	b) $\cos\theta + \sqrt{3}\sin\theta = 1$

c) $12\cos\theta - 5\sin\theta = 6.5$	d) $5\sin\theta - 3\cos\theta = 1$
e) $2\cos 2\theta + 3\sin 3\theta = 3$	f) $8\sin 2\theta - 6\sin 2\theta = 3$

3. Find the Maximum and Minimum of each expression for $0° \le \theta \le 360°$

a) $2\cos\theta + 3\sin\theta$	b) $4\cos\theta - 3\sin\theta$
c) $\sqrt{3}\sin\theta - \cos\theta$	d) $4\sin\theta + 5\cos\theta$

e) $4+3\cos\theta-2\sin\theta$	f) $2\cos\theta+\sin\theta-2$

4. Given that $\cos\theta=\frac{3}{5}$ and θ is in Q1, then what is the value of $\sin\frac{1}{2}\theta$ and $\cos\frac{1}{2}\theta$?

5. Given that $\sin\theta=\frac{-8}{11}$ and θ is in Q4, then what is the value of $\sin\frac{1}{2}\theta$ and $\cos\frac{1}{2}\theta$?

6. Prove the following identities:

$\sin^2\frac{1}{2}\theta=\frac{1}{2}(1-\cos\theta)$	$\cos^2\frac{1}{2}\theta=\frac{1}{2}(1+\cos\theta)$

$\cos\theta = \frac{1-\tan^2\frac{1}{2}\theta}{1+\tan^2\frac{1}{2}\theta}$	$\cot\theta + \csc\theta = \cot\frac{\theta}{2}$

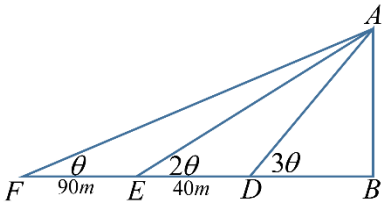
7. Prove the identity: $\frac{\cos x - 5\cos 5x + \cos 9x}{\sin x - 5\sin 5x + \sin 9x} = \cot 5x$

8. Solve for θ, where $0° \le \theta \le 360°$ $\quad \sin 9\theta \sin 8\theta - \sin 7\theta \sin 6\theta = 0$

9. Solve for θ, where $0° \le \theta \le 360°$ $\quad \cos\theta - \cos 3\theta = \sin 2\theta$

10. If $\dfrac{\sin^2\theta}{1+3\cos^2\theta} = \dfrac{3}{16}$, where $90° < \theta < 180°$, find the value of $\dfrac{\sin\theta}{1+3\cos\theta}$

11. AB is a building and BDEF is a level straight line. The angles of elevation of the top of the tower from F, E, D, are $\theta,\ 2\theta,\ and\ 3\theta$ respectively. If $EF = 90m$ and $DE = 40m$ find the height of the building. Hint: The following two identities will be helpful: $[\cot\theta - \cot 2\theta = \csc 2\theta]$ and $\left[\cot 2\theta - \cot 3\theta = \dfrac{1}{2}\sec\theta\csc 3\theta\right]$ (p28 HKCEE).



12. Challenge: Find the general solution of $\tan 5\theta + \cot 2\theta = 0$ (p19 HKCEE)

13. Find the general solution of $\sin 9x \sin 8x - \sin 7x \sin 6x = 0$ HKCE

14. Using the identity: $\cos 3x = \cos(x + 2x)$, prove the following identity: $\cos 3x = 4\cos^2 x - 3\cos x$